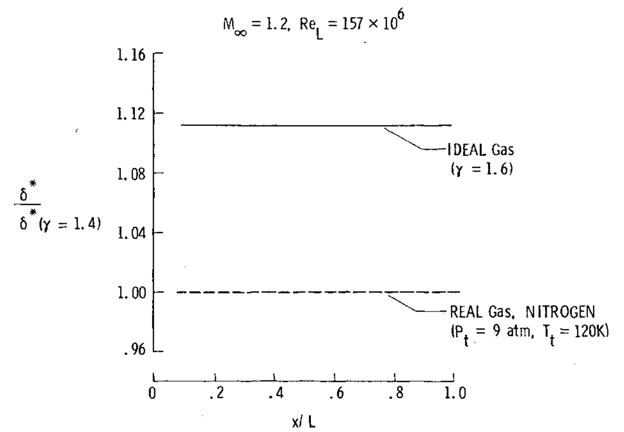

Fig. 2 Relative values of displacement thickness for a turbulent boundary on a flat plate.

The values of relative displacement thickness for the two methods of calculation are shown in Fig. 2. The deviation of the real-gas values is only about 0.1% while the ideal gas $\gamma = 1.6$ values differ by about 11% in the opposite direction. Other boundary-layer parameters, such as momentum and velocity thickness, have deviations for the two methods of calculation which are similar to those for friction coefficient and displacement thickness.

The results indicate that the real-gas effects on a flat-plate turbulent boundary-layer simulation due to testing in a cryogenic nitrogen tunnel are very small and presumably insignificant. Also, these results show that for flat-plate turbulent boundary layers, the use of ideal gas equations in combination with the γ values of cryogenic nitrogen, which in reality are non-constant, gives erroneously high indications of the magnitudes of real-gas effects. Since it has been shown that erroneous results are given for both viscous (this Note) and inviscid flows[3,4] when ideal-gas equations are used in conjunction with real-gas properties, it is extremely doubtful that the large real-gas effects reported by Inger[8,11] for turbulent boundary-layer shock interactions are an accurate prediction of the actual situation that will exist in cryogenic nitrogen gas wind tunnels during the simulation of such flows.

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C80-046

Application of Unsteady Airfoil Theory to Rotary Wings

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An examination of the mathematical models reveals that unsteady airfoil theory is being used incorrectly in almost all major helicopter loads analyses, and also in some aeroelastic stability analyses. The difficulty lies in the identification of the airfoil pitch and heave motions in terms of the variables describing the motion of a rotor blade.

Theodorsen theory for a thin, two-dimensional airfoil undergoing unsteady motion in an incompressible flow gives the following result for the lift and pitch moment:

$$L = \pi\rho b^2(\ddot h + U\dot\alpha - ba\ddot\alpha) + 2\pi\rho UbC(\dot h + U\alpha + b(\tfrac{1}{2} - a)\dot\alpha)$$

$$M = \pi\rho b^2[ba\ddot h - Ub(\tfrac{1}{2} - a)\dot\alpha - b^2(\tfrac{1}{8} + a^2)\ddot\alpha]$$
$$+ 2\pi\rho Ub^2(a + \tfrac{1}{2})C[\dot h + U\alpha + b(\tfrac{1}{2} - a)\dot\alpha]$$

[Eqs. (5-311) and (5-312) of Ref. 1]. Here U is the freestream velocity, b is the airfoil semichord, and ρ is the air density. The airfoil has heave motion h, and α is the pitch angle about the axis a distance ab aft of the midchord. C is the Theodorsen lift deficiency function, which depends on the reduced frequency k. The use of this theory in a helicopter analysis requires that expressions be obtained for h and α in terms of the rotor blade degrees of freedom. To this end, examine the boundary condition of the unsteady airfoil theory, which involves the normal velocity due to the airfoil motion:

$$w_a = -(\dot h + U\alpha) - \dot\alpha(x - ba)$$

[Eq. (5-268) of Ref. 1]. It is only through this boundary condition that the airfoil motion enters the problem. Since the boundary condition depends upon the quantities $(\dot h + U\alpha)$ and $\dot\alpha$, it follows that the solution of this linear problem must depend only upon the same two quantities. Therefore, rewrite the lift and moment as follows:

$$L = \pi\rho b^2[(\dot h + U\alpha)^{\cdot} - ba\ddot\alpha]$$
$$+ 2\pi\rho UbC[(\dot h + U\alpha) + b(\tfrac{1}{2} - a)\dot\alpha]$$

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$$M = \pi\rho b^2 [ba(\dot{h}+U\alpha)^{\cdot} - Ub\tfrac{1}{2}\dot{\alpha} - b^2(\tfrac{1}{8}+a^2)\ddot{\alpha}]$$

$$+2\pi\rho Ub^2(a+\tfrac{1}{2})C[(\dot{h}+U\alpha)+b(\tfrac{1}{2}-a)\dot{\alpha}]$$

Writing the loads in this fashion emphasizes that it is not the airfoil section pitch and heave motions (α and h) that must be identified, but rather the mean and linear components of the normal velocity distribution over the airfoil chord. Observe that the $\dot{\alpha}$ terms actually arise from two sources: from the time derivative of $(\dot{h}+U\alpha)$ as well as from the pitch rate. It is worthwhile noting that in this form the result is applicable as well to the case of time-varying freestream velocity U (see Ref. 2), except that the Theodorsen lift deficiency function must be generalized to account for the expansion and compression of the shed wake vorticity. The result is also applicable for arbitrary motion, again except for the lift deficiency function; indeed, the derivation of Ref. 1 does not find it necessary to introduce the assumption of harmonic motion until the final step of evaluating $C(k)$ [Eq. (5-308) of Ref. 1].

The most common approach in helicopter analyses has been to identify $\dot{h}$ as the normal velocity u_P at the rotor blades, and $\dot{\alpha}$ as the pitch rate $\dot{\theta}+\Omega\beta$ (or sometimes even just $\dot{\theta}$). The numerical results for the lift will not be greatly influenced by this error, since the unsteady lift is small compared to the steady component. The steady moment component is normally small however, so the calculation of the moment will be seriously affected if the unsteady terms are incorrect. Another consequence of the incorrect identification of $\dot{h}$ and α, important for the lift as well as for the moment, is that the equivalence of flapping and feathering motion for an articulated rotor blade will be violated.

As an example, consider the rigid flap and rigid pitch motion of an articulated rotor blade in forward flight. Unsteady airfoil theory requires $(\dot{h}+U\alpha)$, which is the air velocity normal to the airfoil section, at the pitch axis; and $\dot{\alpha}$, which is the equivalent pitch rate or camber of the airfoil. Hence for the present case

$$\dot{h}+U\alpha = u_T\theta - u_P = (\Omega r + \Omega R\mu\sin\psi)\theta - (\Omega R\lambda + r\dot{\beta} + \beta\Omega R\mu\cos\psi)$$

$$\dot{\alpha} = \dot{\theta} + \Omega\beta$$

where Ω is the rotor rotational speed, R is the rotor radius, ψ is the blade azimuth angle, r is the radial location, μ is the advance ratio, and λ is the inflow ratio; an arbitrary reference plane is considered. The flap degree of freedom is β, and θ is the pitch degree of freedom. Substitution of these expressions for $(\dot{h}+U\alpha)$ and $\dot{\alpha}$ give the lift and moment in the form required for helicopter analyses. The components due to the 1/rev flap and pitch motion (using $\lambda = \lambda_{tpp} - \mu\beta_{1c}$, where λ_{tpp} is the inflow ratio relative to the tip-path plane) are:

$$\Delta(\dot{h}+U\alpha) = (\Omega r + \Omega R\mu\sin\psi)\,[(\theta_{1c}-\beta_{1s})\cos\psi + (\theta_{1s}+\beta_{1c})\sin\psi]$$

$$\Delta(\dot{\alpha}) = -(\theta_{1c}-\beta_{1s})\sin\psi + (\theta_{1s}+\beta_{1c})\cos\psi$$

and similarly for their time derivatives. Therefore, even with the unsteady lift and moment included, the equivalence of flapping and feathering motions is maintained. Specifically, the blade loads depend upon the tilt of the tip-path plane relative to the no-feathering plane ($\beta_{1c}+\theta_{1s}$ and $\beta_{1s}-\theta_{1c}$), independent of the orientation of the rotor shaft.

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C80-047

Class of Shockfree Airfoils Producing the Same Surface Pressure

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Introduction

THE design of shockfree airfoils is critical to transonic aerodynamics because shockwaves, which are associated with entropy rises that produce wave drag, also contribute significantly to boundary-layer separation and performance degradation. The existence of shockfree flows is therefore a subject of intense engineering as well as mathematical interest. However, the existence question is many-faceted: different "host" formulations are possible and each provides different information vital to a complete understanding. Taking one approach Morawetz,[1] in the middle 1950's, proved that planar shockfree flows are mathematically isolated in that arbitrary changes to the flow or boundary conditions providing a smooth flow lead to shock formation. This result was recently extended to three dimensions by Cook.[2] On the other hand, Sobieczky[3] and Sobieczky, et al.,[4] recently showed that in two dimensions there exist, for any small change in freestream Mach number, an infinite number of small changes in airfoil shape that will ensure a shockfree flow. In their approach, the required shape changes are selected by a "fictitious gas" scheme embedded in a mixed-type relaxation algorithm[4]; note, however, that the host boundary-value problems used in generating their shockfree airfoils offer no control over the form of the resulting surface pressure distribution. The present paper addresses still another aspect of the general problem, namely, the properties of those airfoils obtained over a wide range of cruise Mach numbers that induce the same fixed supercritical shockfree surface pressure.

The question raised above is interesting for the following reasons. The flow past a fixed airfoil, assuming constant density, is described by a boundary-value problem for the disturbance velocity potential $\phi(x,y)$, namely, $\phi_{xx}+\phi_{yy}=0$, with ϕ_y specified on a chordwise slit, $\nabla\phi\to 0$ at infinity, and a jump in potential $[\phi]$ chosen to satisfy Kutta's requirement for smooth flow from the trailing edge. (x and y are streamwise and transverse coordinates.) On the other hand, the airfoil which produces a prescribed chordwise surface pressure can be found from the solution to a boundary-value problem for the disturbance stream function $\psi(x,y)$. The equation $\psi_{xx}+\psi_{yy}=0$ is solved with ψ_y specified on an approximating slit, noting that $\psi_y=\phi_x=-\tfrac{1}{2}C_p$, where C_p is the surface pressure coefficient, $\psi\to 0$ at infinity, and a jump in stream function $[\psi]=0$ along the downstream slit emanating from the trailing edge to enforce closure (the surface elevations are obtained from $dy/dx=-\psi_x(x,0)$). Because the two formulations are mathematically equivalent, pathological behavior obtained in analysis solutions for ϕ would find their counterpart in design solutions for ψ. Now, for transonic supercritical flow, Morawetz states that small changes in the subsonic freestream Mach number corresponding to a smooth flow lead to discontinuities in surface ϕ_x (unless, of course, the airfoil is simultaneously altered using Sobieczky's procedure); thus, one would suspect that off-design airfoils generated from an inverse procedure

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